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THOMSON AND TAIT'S NATURAL PHILOSOPHY

Treatise on Natural Philosophy. By Sir William Thomson, LL.D., D.C.L., F.R.S., Professor of Natural Philosophy in the University of Glasgow, Fellow of St. Peter's College, Cambridge, and Peter Guthrie Tait, M.A., Professor of Natural Philosophy in the University of Edinburgh, formerly Fellow of St. Peter's College, Cambridge. Vol. I. Part 1. New Edition. (Cambridge, at the University Press, 1879.)

THE year 1867 will long be remembered by natural philosophers as that of the publication of the first volume of "Thomson and Tait." They had long been waiting for the book, and in the preface the delay was accounted for by the necessity of anticipating the wants of the other three volumes, in which the remaining divisions of Natural Philosophy were to be treated. The reader was also reminded, that if in any passage he failed to appreciate the aim of the authors, the reason might be that what he was studying was in reality a prospective contrivance, the true aim of which would not become manifest until after the perusal of that part of the work for which it was designed to prepare the way.

What we have had before us now for twelve years was, the authors reminded us, strictly preliminary matter. The plan of the whole treatise could only be guessed at from the scale on which its foundations were constructed.

In these days, when so much of the science of our best men is dribbled out of them in the fragmentary and imperfectly elaborated form of the memoirs which they contribute to learned societies, and when the work of making books is relegated to professional bookmakers, who understand about as much of one subject as of another, it was something to find that even one man of known power had not shrunk from so great a work; it was more when it appeared that two men of mark were joined together in the undertaking; and when at last the plan of the work was described in the preface, and the scale on which its foundations were being laid was exhibited in the vast substructure of Preliminary Matter, the feeling with which we began to contemplate the mighty whole was one in which delight was almost overpowered by awe.

This feeling has been growing upon us during the twelve years we have been exploring the visible part of the work, marking its bulwarks and telling the rising generation what manner of a palace that must be, of which these are but the outworks and first line of defences, so that now, when we have before us the second edition of the first part of the first volume, we are impelled to risk the danger of criticising an unfinished work, and to say something about the plan of what is already before us.

The first thing which we observe in the arrangement of the work is the prominence given to kinematics, or the theory of pure motion, and the large space devoted under this heading to what has been hitherto considered part of pure geometry. The theory of the curvature of lines and surfaces, for example, has long been recognised as an important branch of geometry, but in treatises on motion it was regarded as lying as much outside of the subject as the four rules of arithmetic or the binomial theorem.

The guiding idea, however, which, though it has long exerted its influence on the best geometers, is now for the first time boldly and explicitly put forward, is that geometry itself is part of the science of motion, and that it treats, not of the relations between figures already existing in space, but of the process by which these figures are generated by the motion of a point or a line

We no longer, for example, consider the line *A B* simply as a white stroke on a black board, and call it indifferently *A B* or *B A*, but we conceive it as the trace of the motion of a point from *A* to *B*, and we distinguish *A* as the beginning and *B* as the end of this trace.

This method of regarding geometrical figures seems to imply that the idea of motion underlies the idea of form, and is in accordance with the psychological doctrine which asserts that at any given instant the attention is confined to a single and indivisible percept, but that as time flows on the attention passes along a continuous series of such percepts, so that the path of investigation along which the mind proceeds may be described as a continuous line without breadth. Our knowledge, therefore, of whatever kind, may be compared to that which a blind man acquires of the form of solid bodies by stroking them with the point of his stick, and then filling up in his imagination the unexplored parts of the surface according to his own notions about continuity and probability. The rapidity, however, with which we make our exploration is such that we come to think that by a single glance we can thoroughly see the whole of that surface of a body which is turned towards us, if, indeed, we are not prepared to assert that we have seen the other side too, when after all, if our attention were to leave a trace behind it, as the point of the blind man's stick might do, this trace would appear as a mere line meandering over the surface in various directions, but leaving between its convolutions unexplored areas, the sum of which is still equal to the whole surface. We are at liberty no doubt to course over the surface and to subdivide the meshes of the network of lines in which we envelope it, and to conclude that there cannot be a hole in it of more than a certain diameter, but no amount of investigation will warrant the conclusion, which, nevertheless we draw at once and without a scruple, that the surface is absolutely continuous and has no hole in it at all. Even when, in a dark night, a flash of lightning discloses instantaneously a whole landscape with trees and buildings, we discover these things not at at once, but by perusing at our leisure the picture which the sudden flash has photographed on our retina.

The reason why the phenomena of motion have been so long refused a place among the most universal and elementary subjects of instruction seems to be, that we have been relying too much on symbols and diagrams, to the neglect of the vital processes of sensation and thought.

It is no doubt much easier to represent in a diagram or a picture the instantaneous relations of things coexisting in space than to illustrate in a full and complete manner the simplest case of motion. When we have drawn our diagram it remains on the paper, and the student may run his mind over the lines in any order which pleases him. But when we are either perceiving real motions, or thinking about them without the aid or the encumbrance of a diagram, the mind is carried along the actual course of the motion, in a manner far more easy and natural than when it is rushing indiscriminately hither and thither along the lines of a diagram.

Having pursued kinematics from its elementary principles till its intricacies begin to be appalling, we resume the study of the elements of science in the opening of the chapter on "Dynamical Laws and Principles." It is here that we first have to deal with something which claims the title of Matter, and our authors, one of whom never misses an opportunity of denouncing metaphysical reasoning, except when he has occasion to expound the peculiarities of the Unconditioned, make the following somewhat pusillanimous statement:—

> "We cannot, of course, give a definition of *Matter* which will satisfy the metaphysician, but the naturalist may be content to know matter as *that which can be perceived by the senses, or as that which can be acted upon by, or can exert, force.*"

The authors proceed to throw out a hint about Force being a direct object of sense, and after telling us that the question *What is matter?* will be discussed in a future volume, in which also the Subjectivity of Force will be considered, they retire to watch the effect of the definition they have thrown into the camp of the naturalists.

Now all this seems to us very much out of place in a treatise on Dynamics. We have nothing of the kind in treatises on Geometry. We have no disquisitions as to whether it is by touch or by sight that we come to know in what way a triangle differs from a square. We have not even a caution that the diagrams of these figures in the book do not exactly correspond with their definitions. Even in kinematics, when our authors speak of the motion of points, lines, surfaces, and solids, though they introduce several modern phrases, the kind of motion they speak of is none other than that which Euclid recognises, when he treats of the generation of figures.

Why, then, should we have any change of method when we pass on from kinematics to abstract dynamics? Why should we find it more difficult to endow moving figures with mass than to endow stationary figures with motion? The bodies we deal with in abstract dynamics are just as completely known to us as the figures in Euclid. They have no properties whatever except those which we explicitly assign to them.

Again, at p. 222, the capacity of the student is called upon to accept the following statement:—

> "Matter has an innate power of resisting external influences, so that every body, as far as it can, remains at rest or moves uniformly in a straight line."

Is it a fact that "matter" has any power, either innate or acquired, of resisting external influences? Does not every force which acts on a body always produce exactly that change in the motion of the body by which its value, as a force, is reckoned? Is a cup of tea to be accused of having an innate power of resisting the sweetening influence of sugar, because it persistently refuses to turn sweet unless the sugar is actually put into it?

But suppose we have got rid of this Manichæan doctrine of the innate depravity of matter, whereby it is disabled from yielding to the influence of a moving force unless that force actually spends itself upon it, what sort of facts are left us to be the subject-matter of abstract dynamics?

We are supposed to have mastered so much of kinematics as to be able to describe all possible motions of points, lines, and figures. In so far as real bodies have figures and motions, we may apply kinematics to them.

The new idea appropriate to dynamics is that the motions of bodies are not independent of each other, but that, under certain conditions, dynamical transactions take place between two bodies, whereby the motions of both bodies are affected.

Every body and every portion of a body in dynamics is credited with a certain quantitative value, called its mass. The first part of our study must therefore be the distribution of mass in bodies. In every dynamical system there is a certain point, the position of which is determined by the distribution of mass. This point was called by Boscovich the centre of mass—a better name, we think, than centre of inertia, though either of these is free from the error involved in the term centre of gravity.

In every dynamical transaction between two bodies there must be something which determines the relation between the alteration of the motions of the two bodies. In other words, there must be some function of the motions of the two bodies which remains constant during the transaction. According to the doctrine of abstract dynamics it is the motion of the centre of mass of the two bodies which is not altered on account of any dynamical transaction between the bodies. This doctrine, if true of real bodies, gives us the means of ascertaining the ratio of the mass of any body to that of the body adopted as the standard of mass, provided we can observe the changes in the motions of the two bodies arising from an encounter between them.

We then confine our attention to one of the bodies, and estimate the magnitude of the transaction between the bodies by its effect in changing the *momentum* of that body, momentum being merely a term for a quantity mathematically defined in terms of mass and motion. The rate at which this change of momentum takes place is the numerical measure of the force acting on the body, and, for all the purposes of abstract dynamics, it *is* the force acting on the body.

We have thus vindicated for figures with mass, and, therefore, for force and stress, impulse and momentum, work and energy, their places in abstract science beside form and motion.

The phenomena of real bodies are found to correspond so exactly with the necessary laws of dynamical systems, that we cannot help applying the language of dynamics to real bodies, and speaking of the masses in dynamics as if they were real bodies or portions of matter.

We must be careful, however, to remember that what we sometimes, even in abstract dynamics, call matter, is not that unknown substratum of real bodies, against which Berkeley directed his arguments, but something as perfectly intelligible as a straight line or a sphere.

Real bodies may or may not have such a substratum, just as they may or may not have sensations, or be capable of happiness or misery, knowledge or ignorance, and the dynamical transactions between them may or may not be accompanied with the conscious effort which the word force suggests to us when we imagine one of the bodies to be our own, but so long as their motions are related to each other according to the conditions laid down in dynamics, we call them, in a perfectly intelligible sense, dynamical or material systems.

In this, the second edition, we notice a large amount of new matter, the importance of which is such that any opinion which we could form within the time at our disposal would be utterly inadequate. But there is one point of vital importance in which we observe a marked improvement, namely, in the treatment of the generalised equations of motion.

Whatever may be our opinion about the relation of mass, as defined in dynamics, to the matter which constitutes real bodies, the practical interest of the science arises from the fact that real bodies *do* behave in a manner strikingly analogous to that in which we have proved that the mass-systems of abstract dynamics *must* behave.

In cases like that of the planets, when the motions we have to account for can be actually observed, the equations of Maclaurin, which are simply a translation of Newton's laws into the Cartesian system of co-ordinates, are amply sufficient for our purpose. But when we have reason to believe that the phenomena which fall under our observation form but a very small part of what is really going on in the system, the question is not—what phenomena will result from the hypothesis that the system is of a certain specified kind? but—what is the most general specification of a material system consistent with the condition that the motions of those parts of the system which we can observe are what we find them to be?

It is to Lagrange, in the first place, that we owe the method which enables us to answer this question without asserting either more or less than all that can be legitimately deduced from the observed facts. But though this method has been in the hands of mathematicians since 1788, when the *Mécanique Analytique* was published, and though a few great mathematicians, such as Sir W. R. Hamilton, Jacobi, &c., have made important contributions to the general theory of dynamics, it is remarkable how slow natural philosophers at large have been to make use of these methods.

Now, however, we have only to open any memoir on a physical subject in order to see that these dynamical theorems have been dragged out of the sanctuary of profound mathematics in which they lay so long enshrined, and have been set to do all kinds of work, easy as well as difficult, throughout the whole range of physical science.

The credit of breaking up the monopoly of the great masters of the spell, and making all their charms familiar in our ears as household words, belongs in great measure to Thomson and Tait. The two northern wizards were the first who, without compunction or dread, uttered in their mother tongue the true and proper names of those dynamical concepts which the magicians of old were wont to invoke only by the aid of muttered symbols and inarticulate equations. And now the feeblest among us can repeat the words of power and take part in dynamical discussions which but a few years ago we should have left for our betters.

In the present edition we have for the first time an exposition of the general theory of a very potent form of incantation, called by our authors the Ignoration of Coordinates. We must remember that the co-ordinates of Thomson and Tait are not the mere scaffolding erected over space by Descartes, but the variables which determine the whole motion. We may picture them as so many independent driving-wheels of a machine which has as many degrees of freedom. In the cases to which the method of ignoration is applied there are certain variables of the system such that neither the kinetic nor the potential energy of the system depends on the values of these variables, though of course the kinetic energy depends on their momenta and velocities. The motion of the rest of the system cannot in any way depend on the particular values of these variables, and therefore the particular values of these variables cannot be ascertained by means of any observation of the motion of the rest of the system. We have therefore no right, from such observations, to assign to them any particular values, and the only scientific way of dealing with them is to ignore them.

But this is not all. Since these variables do not appear in the expression for the potential energy, there can be no force acting on them, and therefore their momenta are, each of them, constant, and their velocities are functions of the variables, but, since their own variables do not enter into the expressions, we may consider them as functions of the other variables, or, as they are here called, the retained co-ordinates, and of the constant momenta of the ignored co-ordinates.

From the velocities as thus expressed, together with the constant momenta, we obtain the contribution of the ignored co-ordinates to the kinetic energy of the system in terms of the retained co-ordinates and of the constant momenta of the ignored co-ordinates. This part of the kinetic energy, being independent of the velocities of the retained co-ordinates, is, as regards the retained co-ordinates, strictly *positional*,[1] and may be considered for all experimental purposes as if it were a term of the potential energy. The other part of the kinetic energy is a homogeneous quadratic function of the velocities of the retained co-ordinates. In the final equations of motion neither the ignored co-ordinates nor their velocities appear, but everything is expressed in terms of the retained co-ordinates and their velocities, the coefficients, however, being, in general, functions of the constant momenta of the ignored co-ordinates.

We may regard this investigation as a mathematical illustration of the scientific principle that in the study of any complex object, we must fix our attention on those elements of it which we are able to observe and to cause to vary, and ignore those which we can neither observe nor cause to vary.

In an ordinary belfry, each bell has one rope which comes down through a hole in the floor to the bellringers' room. But suppose that each rope, instead of acting on one bell, contributes to the motion of many pieces of machinery, and that the motion of each piece is determined not by the motion of one rope alone, but by that of several, and suppose, further, that all this machinery is silent and utterly unknown to the men at the ropes, who can only see as far as the holes in the floor above them.

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[1] The division of forces into motional and positional is introduced at p. 370.

Supposing all this, what is the scientific duty of the men below. They have full command of the ropes, but of nothing else. They can give each rope any position and any velocity, and they can estimate its momentum by stopping all the ropes at once, and feeling what sort of tug each rope gives. If they take the trouble to ascertain how much work they have to do in order to drag the ropes down to a given set of positions, and to express this in terms of these positions, they have found the potential energy of the system in terms of the known co-ordinates. If they then find the tug on any one rope arising from a velocity equal to unity communicated to itself or to any other rope, they can express the kinetic energy in terms of the co-ordinates and velocities.

These data are sufficient to determine the motion of every one of the ropes when it and all the others are acted on by any given forces. This is all that the men at the ropes can ever know. If the machinery above has more degrees of freedom than there are ropes, the co-ordinates which express these degrees of freedom must be ignored. There is no help for it.

Of course, if there are co-ordinates for which there are no ropes, but which enter into the expression for the energy, then, if the motion of these co-ordinates is periodic, there will be "adynamic vibrations" communicated to the ropes, and by these the men below will know that there is something peculiar going on above them. But if they pull the ropes in proper time, they can either quiet these adynamic vibrations or strengthen them, so that in this case these co-ordinates cannot be ignored.

There are other cases, however, in which the conditions for the ignoration of co-ordinates strictly apply. For instance, if an opaque and apparently rigid body contains in a cavity within it an accurately balanced body, mounted on frictionless pivots, and previously set in rapid rotation, the co-ordinate which expresses the angular position of this body is one which we are compelled to ignore, because we have no means of ascertaining it. An unscientific person on receiving this body into his hands would immediately conclude that it was bewitched. A disciple of the northern wizards would prefer to say that the body was subject to gyrostatic domination.

Of the sections on cycloidal motions of systems, we can only here say that the investigation of the constitution of molecules by means of their vibrations, as indicated by spectroscopic observations, will be greatly assisted by a thorough study of this part of the volume.

We have not space to say anything of what to many readers must be one of the most interesting parts of the book—that on continuous calculating machines, in which pure rolling friction is taken from the class of unavoidable evils, and raised to the rank of one of the most powerful aids to science. Rolling and sliding have been more than once combined in the hope of obtaining accurate measurements, but the combination is fatal to accuracy, and these new machines, one at least of which has been actually constructed and used, are the first in which pure rolling friction has had fair play given it as a method of mechanically accurate integration.

A method is also given of combining a number of disk, globe, and cylinder integrators, so as to form a machine the motions of two pieces of which are related to each other by a differential equation of any given form. These machines all work in a purely statical manner, that is, in such a way that the kinetic energy of the system is not an essential element in the practical theory of the machine (as in the case of pendulums, &c.), but has to be taken into account only in order to estimate the magnitude of the tangential forces at the points of contact which might, if great enough, produce slipping between the surfaces. Thus, by means of a machine, which will go as slowly as may be necessary to keep pace with our powers of thought, motions may be calculated, the phases of which in nature pass before us too rapidly to be followed by us.

In the original preface some indications were given of what we were to expect in the remaining three volumes of the work. We hope that the reason why this part of the preface is omitted in the new edition is that the work will now go on so steadily that it will be unnecessary to preface performance by promise.

J. CLERK MAXWELL

ARTIFICIAL MANURES

On Artificial Manures, their Chemical Selection, and Scientific Application to Agriculture. A Series of Lectures given at the Experimental Farm at Vincennes, during 1867 and 1874-5. By M. Georges Ville. Translated and Edited by W. Crookes, F.R.S. (London: Longmans and Co., 1879.)

THOSE who take up this volume with the hope of finding the chemistry of artificial manures fully treated will be much disappointed. Not only are many of the commonest manures scarcely mentioned, but some of the most important and practical aspects of the subject are never noticed. The behaviour of manures after they come in contact with the soil is surely of the greatest importance. Chemical investigations have long ago proved that some of the ingredients of manure—as phosphoric acid and potash—are firmly held in combination by the soil, while others—as nitric acid, chlorine, and soda—are feebly retained, and readily pass away in the drainage water after rain. It has also been abundantly proved that though ammonia is firmly retained by a fertile soil, it rapidly undergoes conversion into nitric acid, which is easily washed out. The practical conclusion from these facts is plain. Diffusible manures must be applied only when the crop can make immediate use of them. Now, though M. Ville speaks voluminously concerning the application of phosphates, nitrates, and ammonium salts, no reference to the facts just indicated is to be found in his book, beyond the mere statement that clay is capable of temporarily retaining potash and ammonia.

The lectures of M. Ville are chiefly occupied by the consideration of the present state of agriculture in France, and by the recommendation of a system of artificial manuring of which he regards himself as the inventor. The condition of French agriculture is clearly, as a whole, very low; the existence of the peasant and small farmer is only maintained by the exercise of much thriftiness and self-denial. To improve this condition M. Ville very